63 4-2

DEPARTMENT OF THE NAVY
OFFICE OF THE CHIEF OF NAVAL OPERATIONS
WASHINGTON 25, D. C.

(OEG)485-63

From: Director, Operations Evaluation Group
To: Distribution List

Subj: Operations Evaluation Group IRM-37; forwarding of

Encl: (1) OEG IRM-37 "Estimation of Takeoff Ground-run Distances for Jet-Propelled Conventional and STOL Aircraft" Unclassified of 19 Apr 1963

1. Enclosure (1) is forwarded for your information and retention.

2. This memorandum presents methods for estimating the takeoff ground-run distances for two types of aircraft which use turbojet or turbofan propulsion systems: conventional takeoff and landing (CTOL) and short takeoff and landing (STOL). Such aircraft are defined as having fixed direction thrust. Only the ground-run phase is analyzed here. Later reports will consider the transition pullup and initial climb phases of the takeoff.

3. Attention is invited to the fact that this type of memorandum does not necessarily represent the opinion of the Operations Evaluation Group or the U. S. Navy.

4. Additional copies of the IRM may be obtained from OEG.

Howard W Kreiner
HOWARD W. KREINER
For the Director
Operations Evaluation Group

DISTRIBUTION:
Attached List

Distribution List for (OEG)485-63:

DDR&E
Dir, WSEG (2)

COM DDC (20)

SNDL

J60 USNA ANNA (2)
J84 SUPTNAVPGSCOL (2)
J95 PRESNAVWARCOL (2)

W6B BUWEPSREP APL/JHU
W7E CONAVAIRDEVCEN
W7C COMNAVAIRTESTCEN

BUWEPS

R-12
R-14
R-55
RA
RAAD-33
RAPP

ONR

405

MARINE CORPS ACTIVITIES

COMDTMARCORPS
Dir, MCLFDC, Quantico
Dir, MCEC

DEPT OF AIR FORCE ACTIVITIES

Dept of AF (8) (Attn: PDO 4008)
USAF Liaison Office, Rand Corp.
C/S, U.S. Air Force (Attn: AFX-PDB)

COMDT, Air University
AFSC

DEPT OF ARMY ACTIVITIES

Dept of Army (Attn: C/R&D) (for RAC)
Dept of Army (Attn: Adj Gen) (6)
COMDT, Army War College

MISCELLANEOUS ACTIVITIES

SRI
U. of Chicago

British Navy Staff
Canadian Joint Staff

DISTRIBUTED PREVIOUSLY TO:

Op09B9
Op09E
Op03B

Op90
Op93
Op34

Op05W
Op52
Op07T

63-4-2

OEG

20

This is an interim report of continuing research. It does not necessarily represent the opinion of OEG or the U. S. Navy. It may be modified or withdrawn at any time.

Interim Research Memorandum

OPERATIONS EVALUATION GROUP

WASHINGTON 25, D. C.

19 April 1963

INTERIM RESEARCH MEMORANDUM

OPERATIONS EVALUATION GROUP

ESTIMATION OF TAKEOFF GROUND-RUN DISTANCES FOR JET-PROPELLED CONVENTIONAL AND STOL AIRCRAFT

by R.D. Linnell

IRM-37

ABSTRACT

This memorandum presents methods for estimating the takeoff ground-run distances for two types of aircraft which use turbojet or turbofan propulsion systems: conventional takeoff and landing (CTOL) and short takeoff and landing (STOL). Such aircraft are defined here as having fixed-direction thrust. The maximum lift coefficient for the landing and takeoff configuration is moderate (around 1.5) for CTOL aircraft but is relatively large (around 3.0) for STOL aircraft. Both types of aircraft can be studied at one time by use of the maximum lift coefficient as a parameter for analysis of takeoff and landing distances. Only the ground-run phase of the takeoff is analyzed here. Later reports will consider the transition pullup and initial climb phases of the takeoff. Correlation of the estimation methods discussed here with experimental data for current and past aircraft will be presented later.

INTRODUCTION

Takeoff is the operation of an aircraft from a condition of rest to controlled flight, usually a climb, at a small altitude of the order of 50 feet. There are three phases to this operation, i.e., the accelerated ground run, an upward-turning transition, and a climb phase, as sketched in figure 1. The takeoff distance is defined as the distance along the surface of the ground to a specified altitude on the flight path. The specified altitude, z_0, is usually 50 feet, but other values are also used.

The ground surface on which the aircraft takes off, indicated in figure 1, is called the runway. It is not absolutely necessary that the ground under the transition and climb phases of the takeoff be a prepared surface, but there must at least be no high obstacles there. The runway is usually very close to horizontal, but some runways may be inclined upward or downward (positive β or negative β) in the direction of takeoff, as indicated in figure 1.

Besides β, three important environmental conditions for takeoff are the character of the runway location, runway surface, and the nature of winds along the runway. The average altitude of the runway can be specified by giving the atmospheric density, whose value also reflects the nature of the climate, i.e., hot, normal, or cold. The runway surface hardness can be given in terms of the coefficient of rolling friction, μ, for wheeled landing gear. It is defined as the ratio of resistive horizontal force to the normal force supported by the wheels. Its value depends on the runway material, the tire pressure, and the speed of the wheel relative to the runway. Approximate, constant values are as follows:

Dry concrete, ice, snow, dry steel	.02
Wet concrete, hard ground	.05
Wet grass, soft sod	.10
Mud, soft sand	.30

The wind speed, V_w, is defined as the runway-component of the speed of the air near the surface of the runway, positive when its direction is opposite to the direction of takeoff. There is usually a wind-gradient, i.e., an increase of wind speed with increasing altitude. Neglect of the gradient is conservative (safe) when there is a positive V_w (headwind), but unconservative for negative V_w (tailwind). The gradient will, however, be neglected here.

The gross characteristics of takeoff that are of greatest interest are the takeoff distance, x_{TOz}, and takeoff time, t_{TOz}. The former gives the runway length and cleared length needed. The time is needed to obtain an estimate of

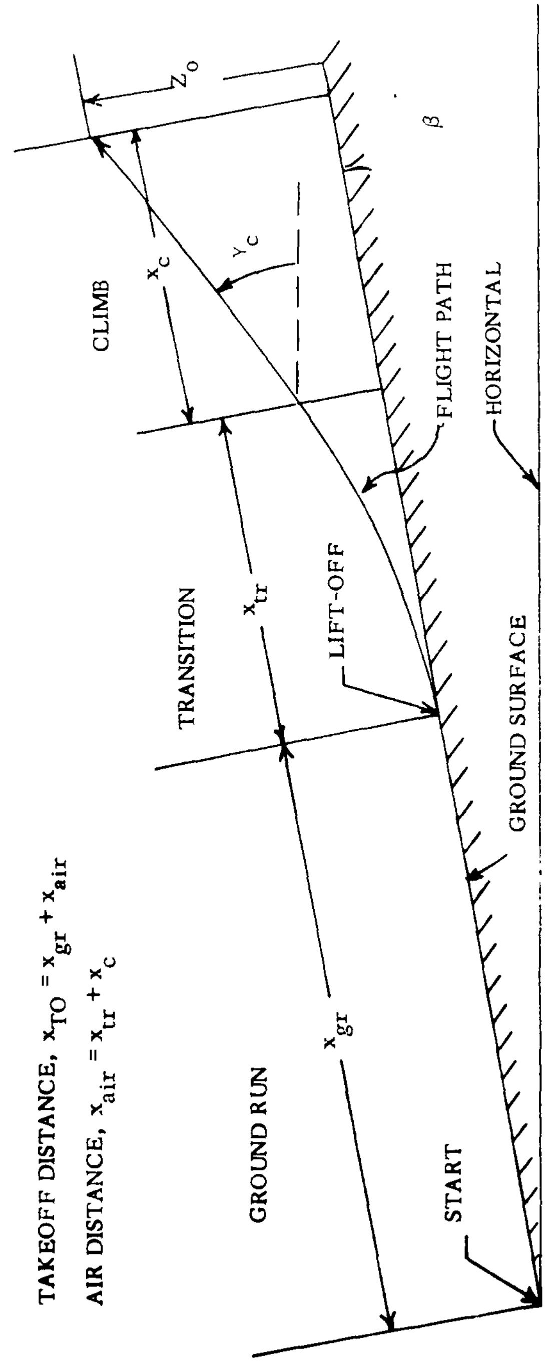

FIG. 1: PHASES OF TAKEOFF OF AN AIRPLANE

the fuel consumed during takeoff. It is desirable to analyze the takeoff in three phases which correspond to the piloting phases. The ground-run distance, x_{gr}, and time, t_{gr}, are obtained first, followed by x_{tr} and t_{tr} for the transition, and x_c and t_c for the climb. These distances are indicated in figure 1, where the sum $(x_{tr} + x_c)$ is designated as the air distance, x_{air}, for convenience of nomenclature. Experimental values for the takeoff characteristics indicate that the exact techniques used by the pilot can cause considerable variation. At best, an accuracy of ±5 percent in the experimental data can be expected. Thus, an analysis of great refinement would not be consistent with the accuracy of the input information (pilot technique) or the possibility of verification (experimental data). On this basis, several approximations will be made in the analysis that follows.

There are several important critical points during the takeoff. Most of them are associated with the operation of mult-engine aircraft and are defined by critical airspeeds, i.e., critical speeds relative to the air. Each such speed can be associated with a distance along the runway, at which the speed occurs during takeoff. These critical distances are indicated in figure 2. If one engine fails, and the remaining engines induce a yawing moment, sufficient aerodynamic and braking moment to maintain control is available only at speeds greater than the Minimum Ground Control speed. If an engine fails at lower airspeeds, the remaining engines must be throttled to avoid an uncontrollable change of ground-path direction.

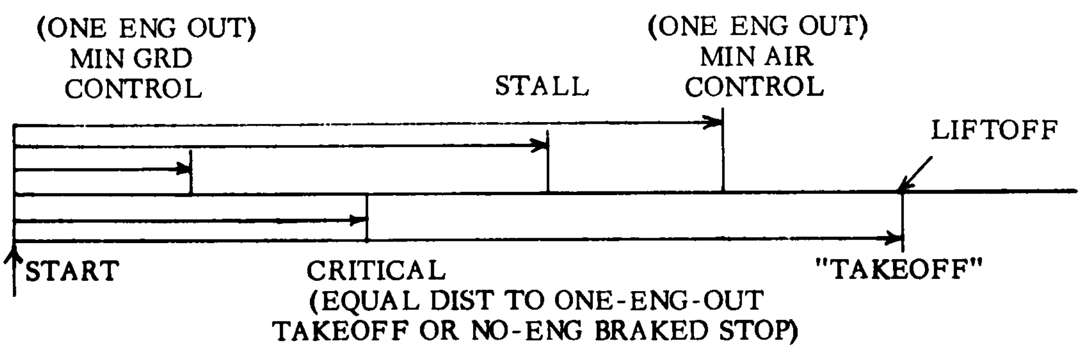

FIG. 2: CRITICAL DISTANCES DURING TAKEOFF

The Critical speed is the speed at which, if an engine fails, the aircraft can takeoff with the remaining engines, to the specified altitude, in the same additional distar ͜e that would be required to stop with full braking after immediately throttling back all the remaining engines. A little thought indicates that for airspeeds less that the Critical speed all engines should be cut when one fails, if the shortest distance to takeoff or stop is desired. However, for airspeeds greater than the critical speed the takeoff should be continued.

The Stall speed is physically well defined, but is subject to some question for estimation purposes. It will be defined here as the speed at which the lift is equal to the takeoff weight, and the maximum lift coefficient in the takeoff configuration, but ignoring ground effect, exists. Thus ground effect and any contribution of engine thrust to lift-direction forces are neglected.

As for ground control, there exists a minimum speed at which aerodynamic moment alone can just overcome yawing moment caused by the failure of one engine. This speed is the Minimum Air Control speed indicated in figure 2. Until this speed is exceeded, the pilot should not terminate the ground run. A somewhat greater speed should be attained before lift-off and the initiation of the transition, to provide a margin of safety with greater controllability and maneuverability. This greater speed at lift-off is called the Takeoff speed.

Hereafter, it will be assumed that all of the critical speeds and distances are less than the stall speed, except the Takeoff speed. The latter is actually the lift-off speed, as described above, but it is conventionally called the Takeoff speed, and denoted by the symbol V_{TO}.

A Reference Ground-Run Distance and Time

An elementary analysis of the takeoff ground-run will provide very simple expressions for the ground-run distance and time. The phenomena that occur are elucidated by this analysis, and the resultant expressions are convenient reference values, which will be denoted by $\tilde{x}_{gr}$ and $\tilde{t}_{gr}$, for later use. The approximations used to obtain these references are to neglect runway slope, runway friction drag, aircraft lift and aircraft drag during the ground run. Then the accelerating force is simply the thrust, T, produced by the propulsion system, and this thrust is approximated as being constant.

With W* denoting aircraft takeoff weight and g denoting the local acceleration of gravity, the acceleration equation becomes

$$\frac{W^*}{g}\frac{d^2x}{dt^2} = T \quad . \tag{1}$$

Direct integration or use of the identity $(d^2x/dt^2) = (1/2)(dV^2/dx)$, where V denotes the flight speed (airspeed), gives

$$
\begin{aligned}
x_2 - x_1 &= (V_2^2 - V_1^2)/2g(T/W^*) \\
t_2 - t_1 &= (V_2 - V_1)/g(T/W^*) \quad .
\end{aligned}
\tag{2}
$$

For the reference parameters with zero wind, the initial conditions are selected as zero, and V_2 becomes the takeoff speed. To provide a margin of safety explicitly, the takeoff speed is taken to be ψ times the stall speed, where ψ is greater than unity. Thus,

$$
V_{TO} = \psi V^*_{st} = \psi\sqrt{2(W^*/A)/\rho C^*_{Lmax}}
\tag{3}
$$

The asterisks are used to emphasize that takeoff conditions are to be used when evaluating the stall speed, aircraft weight and maximum lift coefficient. The safety factor ψ is often assumed to have the value of 1.2 for conventional takeoff and landing (CTOL) aircraft. The effect of variation of its value will be investigated later.

The reference values can now be written as

$$
\begin{aligned}
\tilde{x}_{gr} &= \frac{V_{TO}^2}{2g(T/W^*)} = \frac{\psi^2(W^*/A)}{\rho g(T/W^*)C^*_{Lmax}} = \frac{13\,\psi^2(W^*/A)\text{psf}}{\sigma(T/W^*)C^*_{Lmax}} \ \text{ft} \\
\tilde{t}_{gr} &= \frac{V_{TO}}{g(T/W^*)} = \frac{\psi}{g(T/W^*)}\sqrt{\frac{2(W^*/A)}{\rho C^*_{Lmax}}} = \frac{.9\psi}{(T/W^*)}\sqrt{\frac{(W^*/A)\text{psf}}{C^*_{Lmax}}} \ \text{sec}
\end{aligned}
\tag{4}
$$

where σ denotes the density ratio, ρ/ρ_{SL}, and ρ_{SL} is the standard sea level air density. The first form is basic, but substitution for V_{TO} shows how the aircraft parameters W^*/A and T/W^* affect the ground run distance and time. The final forms are convenient for numerical evaluation. The reference values will be used hereafter as a standard, indicating the basic variation of takeoff distance and time. For example, $\tilde{x}_{gr}$ is linear in wing loading, W^*/A, quadratic in the safety factor ψ, and inversely proportional to σ, thrust-weight ratio and maximum lift coefficient. For variations of aircraft weight only, $\tilde{x}_{gr}$ is proportional to W^{*2}. If the thrust is proportional to air density, then increases of altitude lead to increases of $\tilde{x}_{gr}$ proportional to ρ^{-2} .

If the wind speed, V_w, is not zero, the limits for the airspeed are $V_2 = V_{TO} - V_w$ and $V_1 = V_w$. In terms of the reference values, approximate values for the ground run distance and time, based on constant force by using equation 2, are

$$x_{gr} = \tilde{x}_{gr}\left[1 - \zeta\right]^2$$

$$t_{gr} = \tilde{t}_{gr}\left[1 - \zeta\right] \tag{5}$$

where $\quad \zeta = V_w/V_{TO} = V_w/\psi V_{st}$

The square brackets are correction factors needed to modify the reference values for the effect of wind speed.

Corrections for the Ground-Run Distance and Time

A more complete analysis of the takeoff ground run is needed for new uses, such as analysis of STOL aircraft, analysis of overloaded VTOL aircraft, and operation from unprepared fields in undeveloped countries or in warfare. For modern conventional aircraft, the reference ground-run distance and time provide quite accurate estimates, but a new analytical method is developed below for the newly interesting cases that require consideration of the effects of μ, C_{DO}, C_L and V_w.

The forces acting are sketched in figure 3. The thrust will be considered constant during the ground run, i.e., independent of flight speed, but it can vary with air density. Actually, the direction of action of the thrust depends on its orientation with respect to a zero-lift reference line in the aircraft and the angle of attack of the wing of the aircraft. For the present analysis, the thrust direction is approximated as being parallel to the runway, along the direction of the x-axis, at all times. For small angles this still is a good approximation for the x-component, but there could be a significant z-component even for small angles. Such cases will be considered in later reports, in connection with vertical takeoff and landing aircraft.

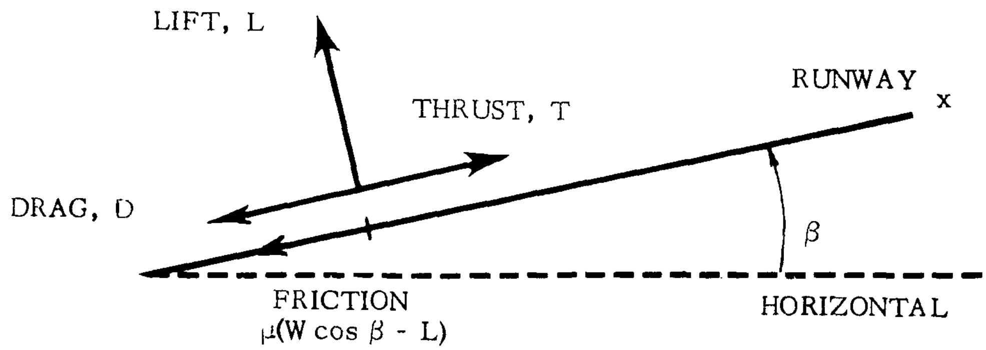

FIG. 3: FORCES DURING TAKEOFF GROUND RUN

For the lift and drag, a symmetrical parabolic polar is used. With asterisks to emphasize that takeoff conditions prevail,

$$L = (1/2)\,\rho V^2 A C_L{}^*$$
$$D = (1/2)\,\rho V^2 A \left[C_{DO}{}^* + K^* C_L{}^{*2} \right] \quad , \tag{6}$$

Here C^*_{DO} is the zero-lift drag coefficient, K^* is the drag-due-to-lift parameter, and V is the flight speed, the sum of the ground speed, V_g, and the runway-component, V_w, of wind speed.

$$V = V_g + V_w \tag{7}$$

The air density, ρ, is essentially constant during the ground run. The lift coefficient, C^*_L, during the ground-run is subject to control manipulation by the pilot. With nose-wheel landing gear the angle of attack is automatically close to zero, and the pilot must exert control forces to obtain a moderate or large angle of attack. For tail-wheel landing-gear the angle of attack is initially large, near the stall angle of attack, and pilot control is required to lift the tail to obtain small angles of attack. An optimum constant lift coefficient exists, as shown on the following page, and will be assumed here.

The flight speeds during takeoff are sufficiently small that C^*_{DO} is very accurately constant, but it is much larger than C_{DO} for clean-configuration flight, because extended landing gear, partial flaps, and possibly external fuel tanks make major contributions. The value of K* may be affected slightly by extended landing gear and ground effect, but it is quite accurately constant.

The acceleration equation for takeoff can be written

$$2\frac{dV_g}{dt} = \frac{dV_g^2}{dx} = 2g\left[\frac{T}{W^*} - \mu\cos\beta - \sin\beta\right] - \frac{\rho g}{(W^*/A)}\left[C^*_{DO} + K^*C_L^{*2} - \mu C^*_L\right](V_g + V_w)^2 \quad . \tag{8}$$

Inspection shows that a small positive lift coefficient increases the acceleration, but that a sufficiently large C^*_L will decrease it. By differentiation of the C^*_L terms it is found that the optimum C^*_L for maximum acceleration is

$$C^*_L \text{ opt} = \mu/2K^* \tag{9a}$$

$$\left[K^*C_L^{*2} - \mu C^*_L\right] \text{opt} = -\mu^2/4K^* \quad . \tag{9b}$$

The drag-due-to-lift parameter, K*, for takeoff is usually small, say approximately .05 for an aspect ratio of about 6. Taking μ equal to .02 for a hard-surfaced runway, the optimum lift coefficient is about .2 or quite moderate. However, for a takeoff from a surface of soft sand, the friction coefficient becomes about .3 and the optimum lift coefficient rises to 3.0, which may not be obtainable. For such cases the correction methods presented below for the takeoff ground run can still be used as described in the next paragraph, even though they are based on operation at the optimum lift coefficient.

For integration, equation 8 can be rewritten using equation 9b and two parameters,

$$\xi = \frac{C^*_{DO} - (\mu^2/4K^*)}{(T/W^*) - \mu\cos\beta - \sin\beta} \quad \frac{\psi^2}{C^*_{Lmax}} \tag{10}$$

$$\zeta = V_w/V_{TO} = V_w/\psi V^*_{st}$$

The parameter ξ can be positive, negative or zero If the optimum lift coefficient is greater than C_{Lmax}, or if the optimum is not used for any reason, the term $\mu^2/4K^*$ in the definition of ξ should be replaced with the actual value of $\left[K^* C_L^{*2} - \mu C_L^*\right]$. If a zero lift coefficient is assumed, then the term should be set equal to zero.

Using the definitions of ξ and ζ, the differential equation becomes

$$\frac{2\,dV_g}{dt} = \frac{dV_g^2}{dx} = 2g\left[\frac{T}{W^*} - \mu\cos\beta - \sin\beta\right]\left[1 - \xi\left(\frac{V_g}{V_{TO}} + \zeta\right)^2\right] \tag{11}$$

The integration must include consideration of the sign of ξ which can be negative or positive. The result can be written in terms of the reference ground run characteristics, with correction factors, as follows.

$$x_{gr} = \frac{(T/W^*)}{(T/W^*) - \mu\cos\beta - \sin\beta} \; \tilde{x}_{gr} \, F(\xi, \zeta) \tag{12}$$

$$\xi \le 0, \text{ then } F = \frac{-1}{\xi} \ln \frac{1 - \xi}{1 - \zeta^2\xi} - \frac{2\zeta}{\sqrt{-\xi}} \left[\tan^{-1}\sqrt{-\xi} - \tan^{-1}\zeta\sqrt{-\xi}\right]$$

$$\xi \geq 0, \text{ then } F = \frac{-1}{\xi} \ln \frac{1-\xi}{1-\zeta^2\xi} \quad \frac{-2\zeta}{\sqrt{\xi}} \left| \tanh^{-1}\sqrt{\xi} - \tanh^{-1}\zeta\sqrt{-\xi} \right|$$

$$t_{gr} = \frac{(T/W^*)}{(T/W^*) - \mu\cos\beta - \sin\beta} \; \tilde{t}_{gr} \; G(\xi, \zeta) \tag{13}$$

$$\xi \leq 0, \text{ then } G = \frac{1}{\sqrt{-\xi}} \left[\tan^{-1}\sqrt{-\xi} - \tan^{-1}\zeta\sqrt{-\xi} \right]$$

$$\xi \geq 0, \text{ then } G = \frac{1}{\sqrt{\xi}} \left[\tanh^{-1}\sqrt{\xi} - \tanh^{-1}\zeta\sqrt{\xi} \right]$$

For specific cases the equations can be evaluated, but, to show the nature and magnitude of the corrections, figures 4 and 5 present contours of constant magnitude of F and G in the ξ, ζ plane. It is seen that the correction factor is within ±10 percent of unity when ζ is close to zero and ξ is in the normal range of $|\xi|$ less than about .25. Thus, in many cases the F and G correction factors can be neglected with fair accuracy. For difficult operating conditions, such as in undeveloped or new countries and for technical guerilla warfare, the corrections can be quite important, and are easily estimated by the use of the thrust-ratio factors of equations 12 and 13 and the functions F and G.

The preceding procedures assume a continuous thrust throughout a continuous takeoff. Discontinuous thrust such as short-duration Jet Assisted Takeoff (JATO) units may be used deliberately, or engine failure may cause a discontinuity of thrust. It is possible for other parameters to vary discontinuously, e.g., the ground-slope angle, β, or the aerodynamic C_{DO}. For such cases the preceding methods could be applied in steps of the ground run. This procedure is fairly simple when drag, friction and lift are ignored, because equations 2 are valid for each step. Use of the method of this section is more complicated. A numerical integration by electronic computer may be simpler in such cases, even though approximations for the variation of C_L^* are required, thus degrading the over-all accuracy of the result. For possible use in such cases the equations for the change of x and t that are obtained by integrating equation 11 between arbitrary limits are given below, where the symbol r denotes the ratio V_g/V_{TO}, and ξ and ζ are as defined in equations 10.

$$x_2 - x_1 = \frac{(V_{TO}^2/2g)}{(T/W^*) - \mu\cos\beta - \sin\beta}\left[F(\xi,\ \zeta + r_1) - F(\xi,\ \zeta + r_2)\right] \quad (14)$$

$$t_2 - t_1 = \frac{V_{TO}/g}{(T/W^*) - \mu\cos\beta - \sin\beta}\left[G(\xi, \zeta + r_1) - G(\xi, \zeta + r_2)\right] \quad (15)$$

The curves of figures 4 and 5 do not extend to large enough values of ζ for general use with these equations, but F and G can be obtained by evaluation of equations 12 and 13.

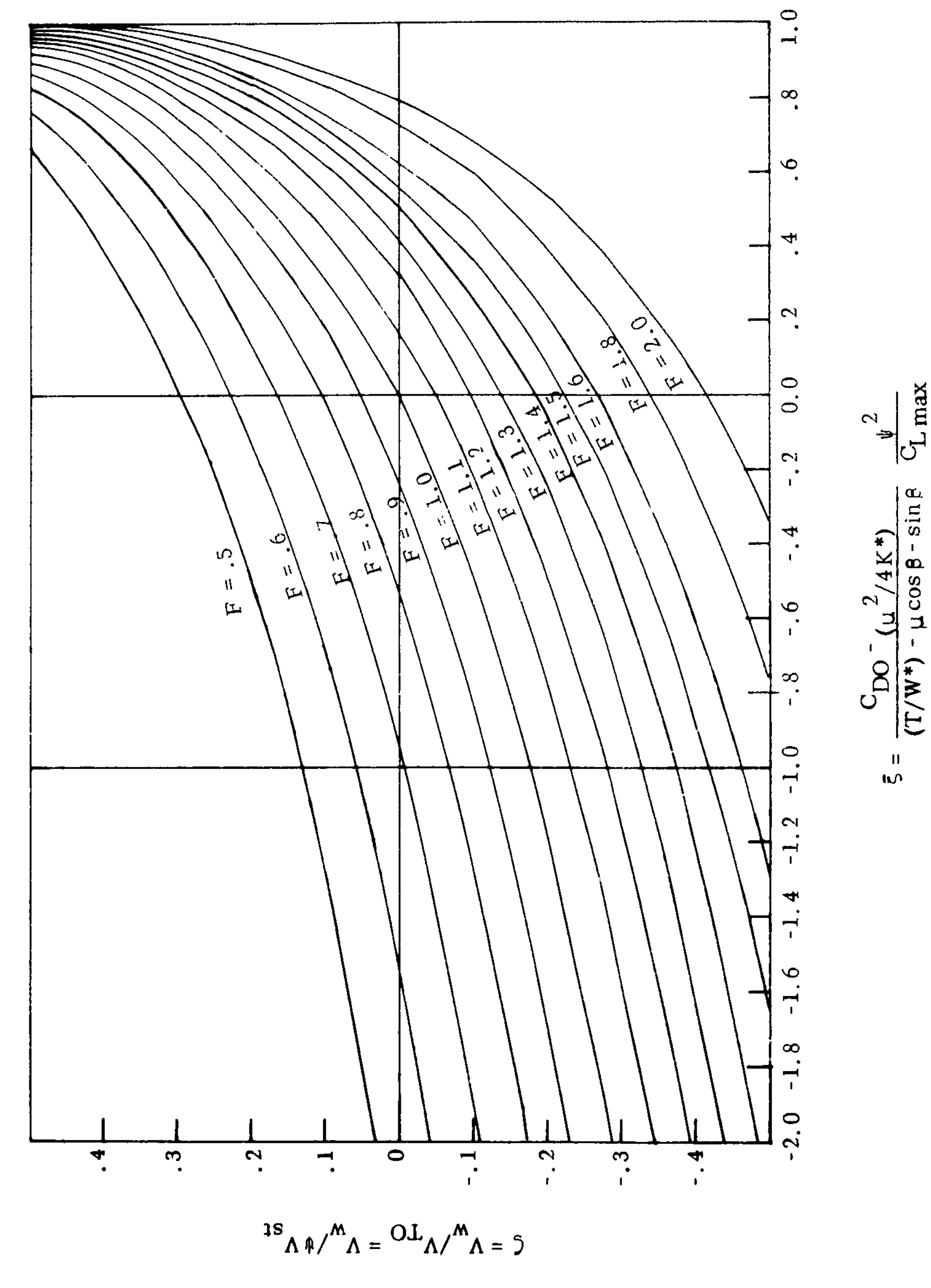

FIG. 4: CORRECTION FACTOR F FOR GROUND-RUN DISTANCE

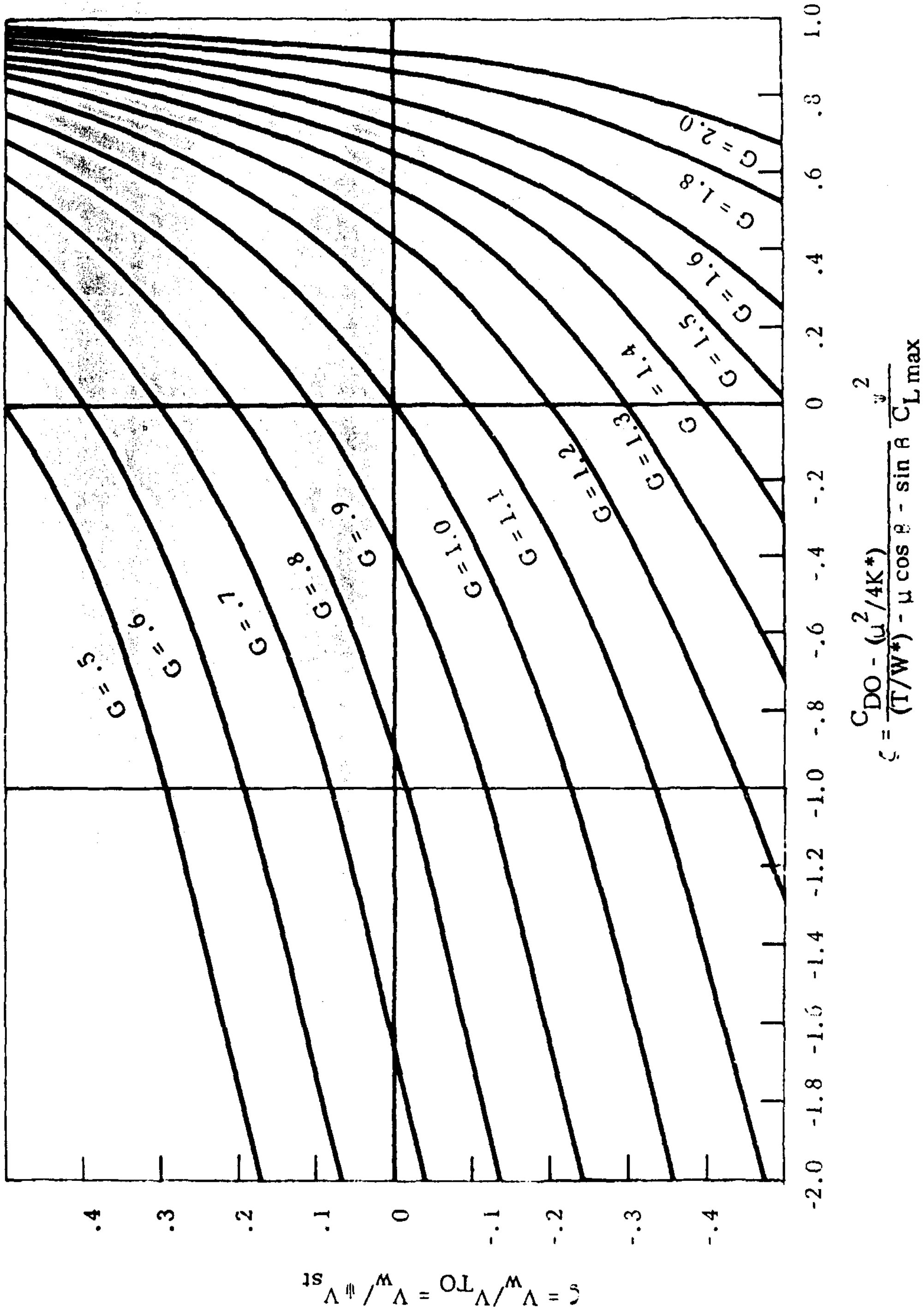

FIG. 5: CORRECTION FACTOR G FOR GROUND-RUN TIME